Analytical Calculation of the Magnetic Vector Potential of an Axisymmetric Solenoid in the Presence of Iron Parts

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This study presents an analytical calculation method for the computation of the magnetic vector potential of an axisymmetric solenoid in the presence of an iron shield and a ferromagnetic core. In this study, the analysis of the current carrying coil in the presence of the ferromagnetic materials is treated as a boundary value problem. The solution approach is based on partitioning the solution domain into distinct regions. The effects of the iron parts are represented by the boundary conditions. The general form of the solution to the Maxwell's equation in each region along with the corresponding boundary conditions is obtained using the Fourier analysis and separation of variables. The continuity of the magnetic vector potential as well as the magnetic field, on the interfaces between the regions, is taken into account by the interface conditions. The final solution to the boundary value problem is constituted by applying the interface conditions on the general solutions to the Maxwell's equations. Finally, the magnetic vector potential is computed over the entire solution domain using the proposed analytical calculation method and the result is compared with the FEM.

***Index Terms*— Boundary value problems, Maxwell equations, Solenoids.**

I. Introduction

The two primary approaches to the modelling of the magnetic field inside the solenoids are the analytical approach and finite element method (FEM). The analytical approach is based on finding an analytical or semi-analytical solution to the electromagnetic boundary value problem [1], [2]. In contrary to the FEM, an analytical approach does not need major modifications to be applied to another system as long as the assumptions inherent in the derivation of the analytical results are not violated. The analytical study of the infinite and finite length solenoids was first performed by calculation of the magnetic field on the solenoid axis [3]. Such studies were followed by the off-axis magnetic field analysis of a solenoid by applying simplifying assumptions to the problem [4], [5]. The final solutions of such studies are presented in terms of the elliptic integrals or the Bessel functions [6]. Most of the studies, in this field, are on the solenoids without an iron core or any ferromagnetic material in the solution domain. Hence, such analytical results cannot simply be extended to the application of the solenoids in the electromagnetic actuators. In electromagnetic actuators (solenoid actuators), the operation of the system is based on the interaction of the current carrying coil and the iron core. Solenoid actuators are widely used in research, industrial and commercial applications. In special applications such as haptics and robotics an accurate model of the magnetic field inside the actuator is required to design a fast and precise model-based closed-loop force control system [7].

The present study follows the boundary value problem approach to provide an expression for the magnetic field inside an electromagnetic solenoid actuator. The presented analytical calculation method is applied on a specific solenoid actuator used in automotive applications and the results are verified by the finite element method (FEM).

II. Model Formulation

The geometry of the electromagnetic actuator studied here is presented in Fig. 1. The system consists of a circular coil of rectangular cross section, an iron shield, a circular ferromagnetic core and air gaps. A cylindrical coordinate is considered by taking the z direction as the symmetry axis. The axial symmetry of the problem implies the independency of the solution to the θ direction thus the solution domain is located inside the rectangular region $0<r<R_3$ and $0<z<z_3$. The solution domain includes those elements involved in the magnetic field generation. The ferromagnetic elements are assumed to have infinite permeability, so the effects of the ferromagnetic elements (core and shield) are taken into account by applying the boundary conditions [7]. As a consequence, the solution domain consists of the coil (Region *I*) and the air gaps (Region *II* and Region *III*).

The solution approach to the magnetic field analysis incorporates the definition of a magnetic vector potential in cylindrical coordinate denoted by $\boldsymbol{A}$ whose curl represents the magnetic flux density (magnetic field) $\boldsymbol{B}$. The axial symmetry of the problem implies that the magnetic vector potential $\boldsymbol{A}$ has only one non-zero component which lies in the θ direction and denoted by A_Θ which is a scalar. Knowing A_Θ, the flux density $\boldsymbol{B} = [B_r, B_\theta, B_z]^T$ can be obtained as follows.

$$B_r(r,z) = -\frac{\partial A_\theta}{\partial z}, B_\theta = 0, B_z(r,z) = \frac{1}{r}\frac{\partial (rA_\theta)}{\partial r} \qquad (1)$$

In a region whose permeability is uniform, the non-zero component of the magnetic vector potential A_Θ is governed by the Poisson's equation as below,

$$\frac{\partial^2 A_\theta}{\partial r^2} + \frac{1}{r}\frac{\partial A_\theta}{\partial r} - \frac{A_\theta}{r^2} + \frac{\partial^2 A_\theta}{\partial z^2} = \nabla^2 A_\theta = -\mu J \qquad (2)$$

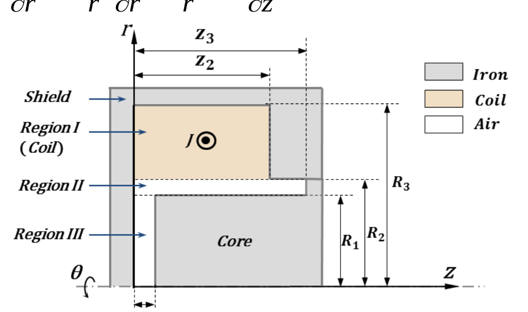
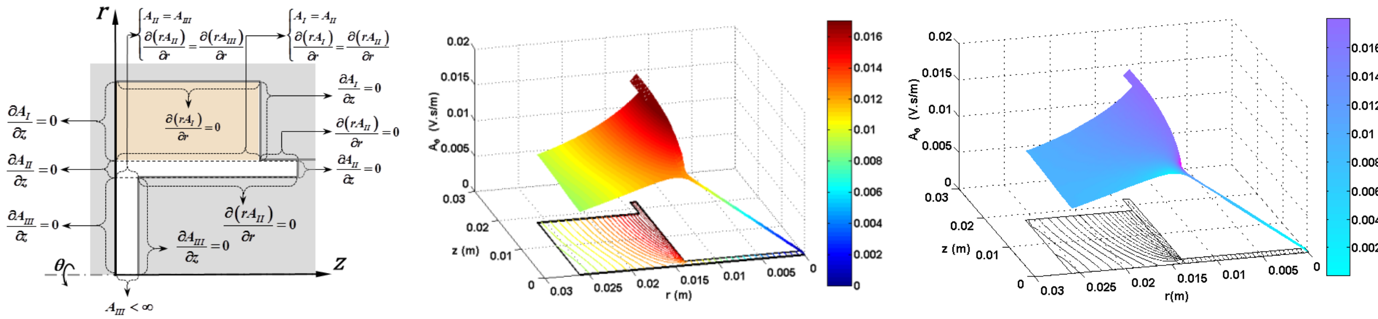


Fig. 1. Geometry of the electromagnetic actuator studied here



(a) (b) (c)

Fig. 2. (a) Boundary and interface conditions (b) Analytical computation result (c) FEM result

where μ is the permeability and J is the current density. When no current passes through a region the Poisson's equation (2) is transformed into the Laplace equation ($\nabla^2 A_\theta = 0$).

The solution domain of the problem is subdivided into three regions which are illustrated in Fig. 1. Hence,

$$\nabla^2 A^I = -\mu_0 J, \;\; \nabla^2 A^{II} = 0, \;\; \nabla^2 A^{III} = 0 \qquad (3)$$

where A_I, A_{II} and A_{III} represent the tangential component of the magnetic vector potential (A_θ) at regions I, II and III, respectively. So it is clear that A_I, A_{II} and A_{III} are scalars.

III. Calculation of the Analytical Solution

The Maxwell's equations (3) at each region together with the boundary conditions applied on that region constitute a boundary value problem. Boundary conditions are the first set of conditions which are applied on the interfaces between the regions and the ferromagnetic materials. The second set of conditions are the interface conditions, which are defined to satisfy the continuity of the magnetic vector potential and the magnetic field in the interfaces between the regions. A summary of the boundary and continuity conditions involved in the solution procedure are presented in Fig. 2(a). The general form of the solution to the boundary value problems is obtained using the separation of variables along with the Fourier analysis. The general form of the solutions for A_I, A_{II} and A_{III} are expressed by infinite series involving cosine and Bessel functions corresponding to the z and r directions, respectively [1]. The general solutions also include the integration constants which must be determined. Assuming that the N harmonic terms of the Fourier series are taken into account (i.e. the general solutions A_I, A_{II} and A_{III} are truncated at the Nth term), the general solutions include $5N+5$ unknown integration constants [7]. Applying the interface conditions (continuity conditions) on the interfaces at $r=R_1$, $r=R_2$ and $r=R_3$ provides $5N+5$ linear equations which are used to calculate the unknown integration constants. Finally, substitution of the integration constants into the general solutions yields the final solution. The accuracy of the final solution is governed by series truncation order (N).

The analytical calculation method proposed in the present study is applied on a linear electromagnetic actuator with known physical properties and the obtained distribution of the magnetic vector potential is presented in Fig. 2(b). The comparison between the proposed analytical computation method and the FEM (Fig. 2(c)) is performed by calculating the absolute value of the error at each node of the FEM mesh.

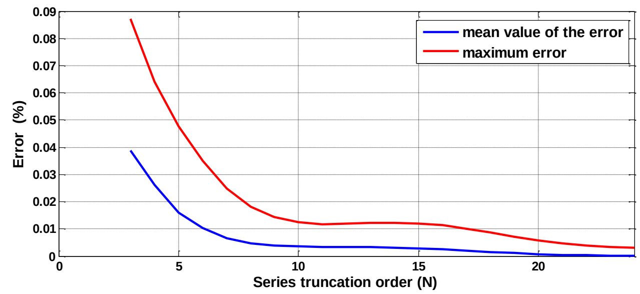


Fig. 3. Error between the result of the presented method and that of the FEM

In Fig. 3, the mean and maximum value of the error is presented as a function of the number of harmonic terms (N) considered in the analytical computation. The good agreement between the results obtained from the proposed method and the FEM confirms the accuracy of the analytical computation approach presented here.

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